TRAVERSE COMPUTATION

Lecture 3

SGU 1053
SURVEY COMPUTATION
 Traverse:

• A series of lines whose lengths and angular relationships have been measured.

Types:

• Closed: Starts and ends at same point (loop) or starts at known point and ends at another known point (link).

• Open: Starts at known or unknown point and ends at unknown point.
Steps to Computation:

1. Adjusting angles or directions to fixed geometric conditions
2. Determining preliminary azimuth (or bearings) of the traverse lines
3. Calculating departures and latitudes and adjust them for misclosures
4. Computing rectangular coordinates of the traverse stations
5. Calculating the lengths and azimuths of the traverse lines after adjustment
1. Balancing Angles:

Apply an average correction to each angle where observing conditions were approximately the same at all stations. The correction is found by dividing the total angular misclosure by the number of angles.

\[
\sum = 540^0 01.2' \\
\sum \text{(interior angles)} = (n - 2) \times 180 \\
\sum = 540^0 00.0' \\
\text{Misclosures} = 1.2'
\]
• Correction: 01.2’/5 = 0.24’
• Subtract correction from each angles

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>100° 44.3’</td>
<td>0.24</td>
<td>0.2</td>
<td>0.2</td>
<td>100° 44.1’</td>
</tr>
<tr>
<td>b</td>
<td>101° 35.1’</td>
<td>0.48</td>
<td>0.5</td>
<td>0.3</td>
<td>101° 34.8’</td>
</tr>
<tr>
<td>c</td>
<td>89° 05.3’</td>
<td>0.72</td>
<td>0.7</td>
<td>0.2</td>
<td>89° 05.1’</td>
</tr>
<tr>
<td>d</td>
<td>17° 11.9’</td>
<td>0.96</td>
<td>1.0</td>
<td>0.3</td>
<td>17° 11.6’</td>
</tr>
<tr>
<td>e</td>
<td>231° 24.6’</td>
<td>1.20</td>
<td>1.2</td>
<td>0.2</td>
<td>231° 24.4’</td>
</tr>
<tr>
<td>Σ</td>
<td>540° 01.2’</td>
<td></td>
<td></td>
<td>1.2’</td>
<td>540° 00.0’</td>
</tr>
</tbody>
</table>
2. Determining Preliminary Azimuth (or Bearings):

But first: The difference between Bearings and Azimuth

**Bearing:**
- The bearing of a line is the direction of the line with respect to a given meridian.
- A bearing is indicated by the quadrant in which the line falls and the acute angle that the line makes with the meridian in that quadrant.
- Observed bearings are those for which the actual bearing angles are measured, while calculated bearings are those for which the bearing angles are indirectly obtained by calculations.
- A true bearing is made with respect to the astronomic north reference meridian.
- A magnetic bearing is one whose reference meridian is the direction to the magnetic poles.
- The location of the magnetic poles is constantly changing; therefore the magnetic bearing between two points is not constant over time.
- The angle between a true meridian and a magnetic meridian at the same point is called its magnetic declination.
Azimuth:

- The azimuth of a line is its direction as given by the angle between the meridian and the line, measured in a clockwise direction.
- Azimuths can be referenced from either the south point or the north point of a meridian.
- Geodetic azimuths traditionally have been referenced to the south meridian whereas grid azimuths are referenced to the north meridian.
- Assumed azimuths are often used for making maps and performing traverses, and are determined in a clockwise direction from an assumed meridian.
- Assumed azimuths are sometimes referred to as "localized grid azimuths."
- Azimuths can be either observed or calculated.
- Calculated azimuths consist of adding to or subtracting field observed angles from a known bearing or azimuth to determine a new bearing or azimuth.
- Azimuths will be determined as a line with a clockwise angle from the north or south end of a true or assumed meridian.
Guidelines:

• Bearings are recorded with respect to its primary direction, north or south, and next the angle east or west.
• A bearing will never be listed with a value over 90 degrees (i.e. the bearing value always will be between over 0 degrees and 90 degrees.
• Bearing angles are computed from a given azimuth depending on the quadrant in which the azimuth lies.
• When the azimuth is in the first quadrant (0° to 90°), the bearing is equal to the azimuth.
• When the azimuth is in the second quadrant (90° to 180°), the bearing is equal to 180° minus the azimuth.
• When the azimuth is in the third quadrant (180° to 270°), the bearing is equal to the azimuth minus 180°.
• When the azimuth is in the fourth quadrant (270° to 360°), the bearing is equal to 360° minus the azimuth.
• Since the numerical values of the bearings repeat in each quadrant, the bearings must be labeled to indicate which quadrant they are in. The label must indicate whether the bearing angle is measured from the north or south line and whether it is east or west of that line.
1st Quadrant

Bearing P = N $42^\circ 16' 38''$ E
Azimuth P = $42^\circ 16' 38''$

2nd Quadrant

Bearing Q = S $32^\circ 24' 46''$ E
Azimuth Q = $147^\circ 35' 14''$

3rd Quadrant

4th Quadrant
• You need a direction of at least one line

Azimuth AT = 234°17.6’
Calculating unknown azimuths: 3 possible cases

- This angle is $-(\alpha_{i-1} + \theta_i - 180^\circ)$, or
- Calculating unknown azimuths: 3 possible cases

(a)

(b)

(c)
Calculating subsequent azimuth:

• Case (a): \( \alpha_i = \alpha_{i-1} + \theta_i - 180^\circ \) \( (i = 0, 1, 2, ...) \)
• Case (b): when \((\alpha_{i-1} + \theta_i - 180^\circ) < 0:\)
  \[
  \alpha_i = (\alpha_{i-1} + \theta_i - 180^\circ) + 360^\circ
  \]
Case (c): when \((\alpha_{i-1} + \theta_i - 180^\circ) > 360^\circ\):
\[\alpha_i = (\alpha_{i-1} + \theta_i - 180^\circ) - 360^\circ\]
<table>
<thead>
<tr>
<th>Line</th>
<th>Computation</th>
<th>Prelim. Azimuth</th>
<th>Prelim. Bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>234° 17.6’ + 151° 52.4’ - 360°</td>
<td>26° 10.0’</td>
<td>N26° 10.0’ E</td>
</tr>
<tr>
<td>BC</td>
<td>26° 10.0’ + (180° - 101° 34.8’)</td>
<td>104° 35.2’</td>
<td>S75° 24.8’ E</td>
</tr>
<tr>
<td>CD</td>
<td>104° 35.2’ + (180° - 89° 05.1’)</td>
<td>195° 30.1’</td>
<td>S15° 30.1’ W</td>
</tr>
<tr>
<td>DE</td>
<td>195° 30.1’ + (180° - 17° 11.6’)</td>
<td>358° 18.5’</td>
<td>N01° 41.5’ W</td>
</tr>
<tr>
<td>EA</td>
<td>358° 18.5’ + (180° - 231° 44.4’)</td>
<td>306° 54.1’</td>
<td>N26° 10.0’ W</td>
</tr>
</tbody>
</table>
3. Calculating Departures (X/E) and Latitudes (Y/N):

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
<td>Departure</td>
<td>Length</td>
</tr>
<tr>
<td>AB</td>
<td>Lat&lt;sub&gt;AB&lt;/sub&gt; = L&lt;sub&gt;AB&lt;/sub&gt; x Cos(Dir&lt;sub&gt;AB&lt;/sub&gt;)</td>
<td>East Dep. and North Lat. is +ve</td>
</tr>
<tr>
<td></td>
<td>Dep&lt;sub&gt;AB&lt;/sub&gt; = L&lt;sub&gt;AB&lt;/sub&gt; x Sin(Dir&lt;sub&gt;AB&lt;/sub&gt;)</td>
<td>West Dep. and South Lat. is -ve</td>
</tr>
</tbody>
</table>
• If all angles and distances are measured perfectly, the $\sum$ of latitude and $\sum$ of departures will be zero
• But errors exist.
• The difference is called departure misclosure and latitude misclosures
• The misclosure will give the indication of precision of measured angles and distances

**Linear Misclosure:**
• Start at a but end up at a’:
Linear misclosure = \( \sqrt{\text{departure misclosure}} + \text{latitude misclosure} \)

Relative precision = \( \frac{\text{linear misclosure}}{\text{traverse length}} \)

<table>
<thead>
<tr>
<th>Order</th>
<th>Max Linear Misclosure</th>
<th>Max Relative Precision</th>
<th>Typical survey task</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>( 2\sqrt{n} )</td>
<td>1 in 25000</td>
<td>Control or monitoring surveys</td>
</tr>
<tr>
<td>Second</td>
<td>( 10\sqrt{n} )</td>
<td>1 in 10000</td>
<td>Engineering surveys; setting out</td>
</tr>
<tr>
<td>Third</td>
<td>( 30\sqrt{n} )</td>
<td>1 in 5000</td>
<td></td>
</tr>
<tr>
<td>Fourth</td>
<td>( 60\sqrt{n} )</td>
<td>1 in 2000</td>
<td>Surveys over small sites</td>
</tr>
<tr>
<td>Station</td>
<td>Prelim. Bearing</td>
<td>Length</td>
<td>Departure</td>
</tr>
<tr>
<td>---------</td>
<td>----------------</td>
<td>--------</td>
<td>-----------</td>
</tr>
<tr>
<td>A</td>
<td>N26° 10.0’ E</td>
<td>285.10</td>
<td>+125.72</td>
</tr>
<tr>
<td>B</td>
<td>S75° 24.8’ E</td>
<td>610.45</td>
<td>+590.77</td>
</tr>
<tr>
<td>C</td>
<td>S15° 30.1’ W</td>
<td>720.48</td>
<td>-192.56</td>
</tr>
<tr>
<td>D</td>
<td>N01° 41.5’ W</td>
<td>203.00</td>
<td>-5.99</td>
</tr>
<tr>
<td>E</td>
<td>N26° 10.0’ W</td>
<td>647.02</td>
<td>-517.40</td>
</tr>
<tr>
<td>A</td>
<td>Σ 2466.05</td>
<td>Σ +0.54</td>
<td>Σ -0.72</td>
</tr>
</tbody>
</table>

\[ \text{Dep}_{AB} = L_{AB} \times \sin(\text{Dir}_{AB}) = 285.10 \times \sin(26° 10.0’) = +125.72 \]

Linear misclosures = \[ \sqrt{\text{departure misclosure}^2 + \text{latitude misclosure}^2} \]

= \[ \sqrt{0.54^2 + 0.72^2} = 0.90 \text{ meter} \]

Relative precision = \[ \frac{\text{linear misclosure}}{\text{traverse length}} = \frac{0.90}{2466.05} = \frac{1}{2700} \]
Traverse Computations and Adjustments
• The linear misclosures must be adjusted (distributed)
• There are a number of methods available for adjusting traverses.
• The most common are listed below:

a. Crandall Rule.
  • The Crandall rule is used when the angular measurements (directions) are believed to have greater precision than the linear measurements (distances).
  • This method allows for the weighting of measurements and has properties similar to the method of least squares adjustment.
  • Although the technique provides adequate results, it is seldom utilized because of its complexity.
  • In addition, modern distance measuring equipment and electronic total stations provide distance and angular measurements with roughly equal precision.
  • Also, a standard Least Squares adjustment can be performed with the same amount of effort.
b. Compass Rule (Bowditch Method)

• The Compass Rule adjustment is used when the angular and linear measurements are of equal precision.
• This is the most widely used traverse adjustment method.
• Since the angular and linear precision are considered equivalent, the angular error is distributed equally throughout the traverse.
• For example, the sum of the interior angles of a five-sided traverse should equal 540° 00' 00".0, but if the sum of the measured angles equals 540° 01' 00".0, a value of 12".0 must be subtracted from each observed angle to balance the angles within traverse.
• After balancing the angular error, the linear error is computed by determining the sums of the north-south latitudes and east-west departures.
• The misclosure in latitude and departure is applied proportional to the distance of each line in the traverse.
c. Least Squares

• The method of least squares is the procedure of adjusting a set of observations that constitute an over-determined model (redundancy > 0).
• A least squares adjustment relates the mathematical (functional model) and stochastic (stochastic model) processes that influence or affect the observations.
• Stochastic refers to the statistical nature of observations or measurements.
• The least squares principle relies on the condition that the sum of the squares of the residuals approaches a minimum.
Traverse Adjustment via Compass (Bowditch Rule)

• The Compass Rule is a simple method and is most commonly employed for engineering, construction, and boundary surveys.
• It is also recognized as the accepted adjustment method in some state minimum technical standards.

• Calculate latitudes (dY or dN) and departures (dX or dE) correction of the traverse misclosure:

\[
dE = \frac{-(total\ departure\ misclosures)}{traverse\ parameter} \times length\ AB
\]

\[
dN = \frac{-(total\ latitude\ misclosures)}{traverse\ parameter} \times length\ AB
\]
• Distribute the misclosure latitudes and departures over the traverse
• Compute adjusted coordinates of the traverse stations
• Calculate final adjusted lengths and azimuths between traverse points

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</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>26° 10.0’</td>
<td>285.10</td>
<td>(-0.06)</td>
<td>+125.72</td>
<td>(+0.08)</td>
<td>255.88</td>
<td>+125.66</td>
<td>+255.96</td>
</tr>
<tr>
<td>BC</td>
<td>104° 35.2’</td>
<td>610.45</td>
<td>(-0.13)</td>
<td>+590.77</td>
<td>(+0.18)</td>
<td>153.74</td>
<td>+590.64</td>
<td>-153.56</td>
</tr>
<tr>
<td>CD</td>
<td>195° 30.1’</td>
<td>720.48</td>
<td>(-0.16)</td>
<td>192.56</td>
<td>(+0.21)</td>
<td>694.27</td>
<td>-192.72</td>
<td>-694.06</td>
</tr>
<tr>
<td>DE</td>
<td>358° 18.5’</td>
<td>203.00</td>
<td>(-0.05)</td>
<td>5.99</td>
<td>(+0.06)</td>
<td>202.91</td>
<td>-6.04</td>
<td>+202.97</td>
</tr>
<tr>
<td>EA</td>
<td>306° 54.1’</td>
<td>647.02</td>
<td>(-0.14)</td>
<td>517.40</td>
<td>(+0.19)</td>
<td>388.50</td>
<td>-517.54</td>
<td>+388.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>+716.49</td>
<td>-715.95</td>
<td>+847.29</td>
<td>-848.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>misclosure</td>
<td>+0.54</td>
<td>misclosure</td>
<td>-0.72</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

misclosure
4. Computing Rectangular Coordinates:

- You need coordinates to compute:
  - Length and directions of lines
  - Area
  - Curve calculations
  - Locating inaccessible points
  - Ease in plotting maps

\[ X_B = X_A + \text{departure AB} \]
\[ Y_B = Y_A + \text{latitude AB} \]

<table>
<thead>
<tr>
<th>Point</th>
<th>Coordinates</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X (Easting)</td>
<td>Y (Northing)</td>
</tr>
<tr>
<td>A</td>
<td>10000.00</td>
<td>10000.00</td>
</tr>
<tr>
<td>B</td>
<td>10125.66</td>
<td>10255.96</td>
</tr>
<tr>
<td>C</td>
<td>10716.30</td>
<td>10102.40</td>
</tr>
<tr>
<td>D</td>
<td>10523.58</td>
<td>9408.34</td>
</tr>
<tr>
<td>E</td>
<td>10517.54</td>
<td>9611.31</td>
</tr>
</tbody>
</table>
Computing Lengths and Azimuth/Bearings from Departure/Latitudes:

\[ \text{tan azimuth (or bearing)} = \frac{\text{departure}}{\text{latitude}} \]

\[ \text{length} = \frac{\text{departure}}{\sin \text{azimuth (or bearing)}} = \frac{\text{latitude}}{\cos \text{azimuth (or bearing)}} \]

\[ = \sqrt{\text{departure}^2 + \text{latitude}^2} \]

- The process of computing lengths and directions from departures and latitudes or from coordinates is called inversing
5. Calculating the lengths and azimuths of the traverse lines after adjustment:

- Used the adjusted departures and latitudes to compute adjusted azimuth and lengths

<table>
<thead>
<tr>
<th>Line</th>
<th>Balanced Length</th>
<th>Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>285.14</td>
<td>26°08.9'</td>
</tr>
<tr>
<td>BC</td>
<td>610.28</td>
<td>104°34.4'</td>
</tr>
<tr>
<td>CD</td>
<td>720.32</td>
<td>195°31.1'</td>
</tr>
<tr>
<td>DE</td>
<td>203.06</td>
<td>358°17.6'</td>
</tr>
<tr>
<td>EA</td>
<td>647.25</td>
<td>306°54.5'</td>
</tr>
</tbody>
</table>

$$\tan \text{azimuth}_{AB} = \frac{125.66}{255.96} = 0.490936, \text{azimuth}_{AB} = 26°08.9'$$

$$\text{length}_{AB} = \sqrt{25.66^2 + 55.96^2} = 285.14 \text{ meter}$$